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THE SPATIAL CONTACT PROBLEM FOR AN ELASTIC WEDGE WITH UNKNOWN CONTACT AREA†

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The contact problem of the indentation of an elliptic paraboloid into one side of a spatial wedge, the other side of which is free from stresses, is investigated without introducing any limitations on the remoteness of the punch from the edge of the wedge and on the aperture angle of the wedge. In the case when the punch approaches close to the edge, the method of non-linear boundary equations of the Hammerstein type is used [1, 2], which enables the normal contact pressures and the unknown contact area to be determined simultaneously. The kernel of the integral equation of the contact problem is then regularized both outside the edge and on the edge of the wedge. The solution obtained agrees well with that obtained in [3], constructed using the asymptotic "large λ" method, which is effective when the punch is sufficiently far from the edge of the wedge, when the contact area can be assumed to be an ellipse, and also with the exact solution of the corresponding contact problem for a half-space [4]. A numerical analysis of the asymmetry of the contact area, the dependence of the indenting force on the settling of the punch, and the effective stresses at the point of initial contact for different aperture angles of the wedge and two orientations of the elliptic paraboloid with respect to the edge is carried out for values of the parameters of the problem given in [5].

The method of finite elements was used previously in [6] to investigate the contact problem for a quarter of space for a rectangular contact area.

1. Suppose that a rigid punch whose surface is an elliptic paraboloid $f(r, z) = (r-a)^2/(2R_1) + z^2/(2R_2)$ is pressed into the side $\varphi = \alpha$ of an elastic three-dimensional wedge with aperture angle α (r, φ and z are the cylindrical coordinates and the z axis coincides with the edge of the wedge) by a force P applied along the r axis at a distance H from the edge. Due to the action of the force P applied along the r axis at a distance H from the edge. Due to the action of the force P the punch settles an amount δ and rotates through an angle γ about the straight line $r = a$. Outside the contact area there is no load on the side $\varphi = \alpha$. We neglect the friction forces between the wedge and the punch. The side $\varphi = 0$ is assumed to be stress-free. It is required to determine, for specified values of $\delta, \gamma, a, R_1, R_2$, the contact area Ω, the distribution function of the normal contact stresses $\sigma_\varphi(\rho, \alpha, z) = -q(r, z)$ ($(r, z) \in \Omega$), and also the quantities P and H.

We will assume that the area Ω is completely contained within a rectangle S with centre on the r axis and semiaxes b and c ($b \geqslant c$). The integral equation and inequality, to which the solution of this problem can be reduced, have the form [1, 3] (where G is the shear modulus and ν is Poisson's ratio)

$$\begin{aligned} &\theta\int_S K(M,N)q(N)d\Omega_N = g(M); \quad q(M)\geqslant 0, \quad M\in\Omega \\ &\theta\int_S K(M,N)q(N)d\Omega_N > g(M); \quad q(M)=0, \quad M\in(S\setminus\Omega) \end{aligned} \tag{1.1}$$

$$M=(r,z),\quad N=(x,y),\quad \theta=(1-\nu)/G,\quad g(r,z)=2\pi(\delta+\gamma(r-a)-f(r,z))$$

$$K(r,z,x,y)=1/R+F(r,z,x,y),\quad R=\sqrt{(r-x)^2+(z-y)^2}$$

$$\begin{aligned} F(r,z,x,y)=\frac{4}{\pi^2}\int_0^\infty\int_0^\infty\Big\{&\operatorname{sh}\pi u(W(u)-\operatorname{cth}\pi u)K_{iu}(\beta x)+ \\ &+\operatorname{sh}\frac{\pi u}{2}[W_+(u)F_+(u,\beta x)-W_-(u)F_-(u,\beta x)]\Big\}K_{iu}(\beta r)\cos\beta(z-y)\,d\beta du \end{aligned} \tag{1.2}$$

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$$F_\pm(u,\beta x)=(1-2\nu)\int_0^\infty L_\pm(u,y)[F_\pm(y,\beta x)+\operatorname{ch}\frac{\pi y}{2}K_{iy}(\beta x)]dy,\quad 0\leq u<\infty \tag{1.3}$$

$$L_\pm(u,y)=2\operatorname{ch}\frac{\pi u}{2}\operatorname{sh}\frac{\pi y}{2}W_\pm(y)\int_0^\infty\frac{\operatorname{sh}\pi t g_\pm(t)dt}{(\operatorname{ch}\pi t+\operatorname{ch}\pi u)(\operatorname{ch}\pi t+\operatorname{ch}\pi y)}$$

$$W_\pm(u)=\pm\frac{\operatorname{ch}\alpha u\mp\cos\alpha}{\operatorname{sh}\alpha u\pm u\sin\alpha},\quad W(u)=\frac{W_+(u)-W_-(u)}{2},\quad g_\pm(t)\leftrightharpoons\left(\operatorname{cth}\frac{\alpha t}{2}\right)^{\pm1}\frac{\sin^2\alpha}{\operatorname{ch}\alpha t\mp\cos 2\alpha}$$

It is also assumed that a bounded region $S_0 = \{M: g(M) > 0\}$ exists such that $\Omega \subset \bar{S}_0 \subset S$. We will introduce the non-linear operators [1, 2]

$$\upsilon^+(M)=\sup\{\upsilon(M),0\},\quad \upsilon^-(M)=\inf[\upsilon(M),0\}$$

and consider the operator equation

$$T\upsilon=0\ (M\in S),\quad T\upsilon\equiv\mu\upsilon^-+\theta K\upsilon^+-g,\quad \mu=\text{const} \tag{1.4}$$

where $\upsilon^\pm = \upsilon^\pm(M)$, $g = g(M)$ and K is an integral operator of the form

$$K\upsilon^+=\int_S K(M,N)\upsilon^+(N)dS_N \tag{1.5}$$

Theorem 1. If $\upsilon_* = \upsilon_*(M)$ is the solution of Eq. (1.4), then $(q = q(M) = \upsilon_*^+, \Omega = \{M: \upsilon_* \geq 0\})$ is the solution of Eq. (1.1), where $\Omega \neq \phi$ when $S_0 = \phi$; conversely, if (q, Ω) is the solution of Eq. (1.1), then $\upsilon_* = \mu^{-1}g + q - \theta\mu^{-1}Kq$, $M \in S$ is the solution of Eq. (1.4).

Theorem 2. For a unique solution $\upsilon_* \in L_2(S)$ of Eq. (1.4) to exist it is necessary and sufficient that the function $\upsilon_0 = \upsilon_0(M)$, which serves as the solution of the equation $(\varepsilon_* > 0)$

$$\varepsilon_*\upsilon^++\mu\upsilon^-+\theta K\upsilon^+=g\quad (M\in S) \tag{1.6}$$

should satisfy the condition

$$\|\upsilon_0\|_{L_2}\leq C,\quad \varepsilon_*\in(0,\varepsilon_0],\quad \varepsilon_0=\text{const}>0$$

where the constant C is independent of ε_*.

Equation (1.6) has a solution by virtue of the principle of contractive mappings for sufficiently large values of μ [2].

Theorem 3. Suppose $\upsilon_1(M)$ and $\upsilon_2(M)$ are the solutions of Eq. (1.4) when $\mu = \mu_1$ and $\mu = \mu_2$, respectively $(\mu_1 \neq \mu_2)$. Then $\upsilon_1^+(M) = \upsilon_2^+(M)$.

The proofs of these three theorems, which are key theorems for the method of non-linear boundary equations, repeat the proofs of the corresponding theorems in [1]. Here we use the fact that the integral operator K of the form (1.5) is completely continuous, self-conjugate, strictly positive, and its kernel $K(M, N)$ possesses a weak singularity.

To determine the quantities P and H we must add to Eq. (1.4) the following two integral equations of equilibrium

$$\int_\Omega q(M)d\Omega_M=P,\quad \int_\Omega rq(M)d\Omega_M=PH \tag{1.7}$$

For a numerical solution of Eq. (1.4) we will use Krasnosel'skii's method [7], which is based on the construction of successive approximations using the formulae

$$\upsilon_{n+1}=\upsilon_n-(Q'\upsilon_n)^{-1}T\upsilon_n \tag{1.8}$$

$$\upsilon_n=\upsilon_n(M),\quad n=0,1,2...,\quad \upsilon_0=g$$

where Q is a differentiable operator which approximates the operator T of the form (1.4) quite well in a uniform metric, and has the form†

$$Q\upsilon = \mu(\upsilon - Q_1\upsilon) + \theta K Q_1 \upsilon - g$$

$$Q_1\upsilon = \begin{cases} 0, & \upsilon < -\varepsilon_1 \\ \frac{1}{2}(\upsilon - \frac{1}{2}\upsilon^2/\varepsilon_1) + \frac{3}{4}\varepsilon_1, & |\upsilon| \leqslant \varepsilon_1 \\ \upsilon, & \upsilon > \varepsilon_1 > 0 \end{cases} \tag{1.9}$$

where, by choosing the constant ε_1, we can endeavour to approximate the operator T with any accuracy specified in advance.

In view of the symmetry of the problem with respect to z it is sufficient to consider solely the upper half of the rectangle S, which we will cover with a net of m nodes with spacing h_1 along the r axis and h_2 along the z axis (in the calculations $m = 81$). When calculating the values of the function $K(M, N)$ of the form (1.2) at these nodes its singularity outside the edge of the wedge is smoothed using the formulae

$$1/R \to 1/R_*, \quad R_* = \sqrt{(r-x)^2 + (z-y)^2 + \delta_*} \tag{1.10}$$

and on the edge

$$K(0,z,x,y) \to A_0/R_0, \quad R_0 = \sqrt{x^2 + (z-y)^2 + \delta_*} \tag{1.11}$$

$$A_0 = \pi\frac{2\alpha + \sin 2\alpha}{2(\alpha^2 - \sin^2\alpha)} + \frac{2}{\pi}\int_0^\infty \operatorname{th}\frac{\pi u}{2}\{W_+(u)F_+(u) - W_-(u)F_-(u)\}\times$$

$$\times\cos[u\ln((R_0 + |z-y|)/(x + \sqrt{\delta_*}))]du$$

$$F_\pm(u) - (1-2\nu)\int_0^\infty L_\pm(u,y)F_\pm(y)dy = \frac{\pi}{2}(1-2\nu)L_\pm(u,0), \quad 0 \leqslant u < \infty \tag{1.12}$$

$$L_\pm(u,0) = \pm\pi\frac{1\mp\cos\alpha}{\alpha\pm\sin\alpha}\operatorname{ch}\frac{\pi u}{2}\int_0^\infty \operatorname{th}\frac{\pi t}{2}\frac{g_\pm(t)}{\operatorname{ch}\pi t + \operatorname{ch}\pi u}dt$$

When deriving (1.11) and (1.12) we took into account the fact that $K_{iu}(0) = \pi\delta(u)$ (δ is the Dirac function). It can be shown that the regularizing parameter in (1.10)–(1.12) must be related to the net spacings h_1 and h_2 (we assumed $\delta_* = h_1h_2/16$ in the calculations).

2. If the rectangle S does not reach the edge of the wedge, we place its centre at the point $r = a$, $z = 0$ and introduce the following dimensionless quantities and notation

$$r - a = r'b, \quad x - a = x'b, \quad z = z'b, \quad y = y'b, \quad \delta = \delta'b, \quad H = H'b$$
$$A = b/(2R_1), \quad B = b/(2R_2), \quad \lambda = a/b, \quad \varepsilon = c/b \tag{2.1}$$
$$\theta q(r,z) = 2\pi q'(r',z'), \quad \theta P = 2\pi b^2 P', \quad S' \to S, \quad \Omega' \to \Omega$$

It is obvious that formulae (2.1) hold when $\lambda > \varepsilon$ when the rectangle S is elongated along the z axis ($R_1 \leqslant R_2$) or $\lambda > 1$ when it is elongated along the r axis ($R_1 \geqslant R_2$). When $\lambda \leqslant \varepsilon$ ($R_1 \leqslant R_2$) or $\lambda \leqslant 1$ ($R_1 \geqslant R_2$) we will assume that one side of the rectangle is situated on the edge of the wedge, and we will take as its centre the point $r = c, z = 0$ when $R_1 \leqslant R_2$ or the point $r = b, z = 0$ when $R_1 \geqslant R_2$. In these cases we will also use the notation (2.1), replacing the first two equations of (2.1) by $r - c = r'b, x - c = x'b$ when $R_1 \leqslant R_2$ or $r - b = r'b, x - b = x'b$ when $R_1 \geqslant R_2$. We will henceforth omit the primes.

The dimensionless parameter λ represents the relative remoteness of the punch from the edge of the wedge.

†GALANOV B. A. The method of non-linear boundary equations in contact problems with unknown contact areas. Doctorate dissertation, Kiev, 1987.

To debug the computer program which we developed we used the following:

(a) the exact solution of the axisymmetric contact problem [8], which, when $\alpha = \pi$, $\gamma = 0$, $\varepsilon = \delta = A = B = 1$ (λ and ν can have any values) has the form

$$q(r,z) = q_0\sqrt{1-(r/c_1)^2-(z/c_2)^2}$$
$$q_0 = 2\sqrt{2}/\pi^2, \quad c_1 = c_2 = 1/\sqrt{2} \tag{2.2}$$

(b) the exact solution of the problem of the indentation of an elliptic paraboloid into a half-space [4] ($\alpha = \pi$, $\varepsilon = 0.5$, $\delta = 1$, $\gamma = 0$, $A = 2$, $B = 1$; λ and ν can have any values), defined by formula (2.2) for $q_0 = 0.348$, $c_1 = 0.469$ and $c_2 = 0.744$;

(c) the solution obtained using the asymptotic "large λ" method [3] ($\alpha = \pi/2$, $\nu = 0.3$, $\lambda = 2$, $\varepsilon = 0.5$, $\delta = 1$, $\gamma = -0.0450$, $A = 2.19$ and $B = 1$), which is found from (2.2) with $q_0 = 0.326$, $c_1 = 0.422$ and $c_2 = 0.704$.

The results of a comparison of solutions (a)–(c) at nine nodes on the r axis with the corresponding values obtained using the computer program show that the difference in cases (a) and (b) is less than 3%, while in case (c) it does not exceed 12%.

Values of the indenting force 10^3P as a function of the settling of the punch $10^3\delta$ are given in Table 1 for different values of α and two orientations of the punch with respect to the edge. Here $\nu = 0.3$, $\lambda = 0$, $\varepsilon = 0.15$, $\gamma = 0$ and $A = 0.005$ and $B = 0.1$ (up to values of $\alpha = 180°$ inclusive) or $A = 0.1$ and $B = 0.005$ (below the row corresponding to $\alpha = 180°$); for $\alpha = 180°$ we give the exact values [4]. In view of the regularities of (1.11) and (1.12) the value $\lambda = 0$ corresponds to the case when the point at which the punch and the wedge initially touch "almost" reaches the edge. An analysis of these results, and also of the corresponding calculations, carried out for $\varepsilon = 0.1$ and $\varepsilon = 0.25$ shows that when $\alpha \approx 90°$ the value of $P = P(\delta)$ as $\lambda \to 0$ is independent of which axes of coordinates (r or z) the elliptic paraboloid is elongated along.

In Figs 1 and 2 the upper half of the contact areas Ω are shown hatched for angles $\alpha = 65°$ (Fig. 1) and $\alpha = 135°$ (Fig. 2). Here $\nu = 0.3$, $\varepsilon = 0.15$, $\delta = 0.005$, $\gamma = 0$, $A = 0.1$ and $B = 0.005$; for $\lambda = 0$ the boundary of the region Ω is shown by the continuous curve, while for $\lambda = \varepsilon$ it is shown by the dashed curve. It can be seen that for $\alpha = 65°$ the area of the region Ω is considerably less than for $\alpha = 135°$ (this also occurs in the case when $A < B$). For fairly acute angles α and $\lambda \to 0$ breakdown of the contact is observed in the neighbourhood of the point where the punch and the wedge initially touch (as though the edge withdraws), particularly when the punch is elongated along the edge (Fig. 1).

3. After solving the contact problem, knowing the function $q(r, z)$ and the contact area Ω, it is possible to determine the effective dimensionless stress $\sigma_e' = \theta\sigma_e/(2\pi)$, which plays an important role in applications. As an example (in the framework of the concept of surface strength) we will determine σ_e' at the point where the punch and the wedge initially touch $r = a_0$, $\vartheta = \alpha$, $z = 0$ ($a_0 = \lambda - \varepsilon$ when $\lambda \leqslant \varepsilon$ and $R_1 \leqslant R_2$, $a_0 = \lambda - 1$ when $\lambda \leqslant 1$ and $R_1 \geqslant R_2$, while in other cases $a_0 = 0$) from the following formulae (we omit the primes)

$$\sigma_e = 2^{-1/2}[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2]^{1/2} \tag{3.1}$$

Table 1

α, deg	$10^3\delta = 4$	4.5	5	5.5	6	6.5
65	0.116	0.141	0.167	0.194	0.222	0.250
90	0.210	0.255	0.302	0.351	0.400	0.453
110	0.253	0.306	0.361	0.419	0.478	0.541
135	0.311	0.374	0.441	0.510	0.581	0.657
180	0.601	0.717	0.840	0.969	1.10	1.24
135	0.440	0.527	0.616	0.713	0.812	0.913
110	0.314	0.376	0.441	0.509	0.581	0.654
90	0.215	0.257	0.302	0.348	0.396	0.447
65	0.0813	0.0967	0.113	0.131	0.150	0.168

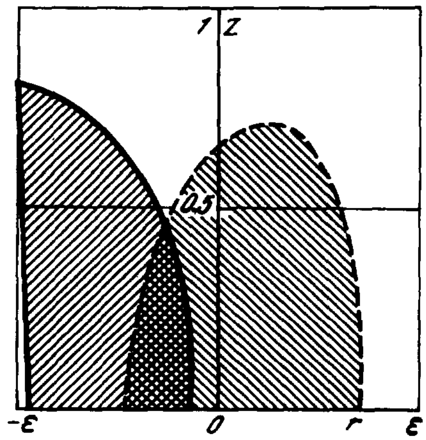

Fig. 1.

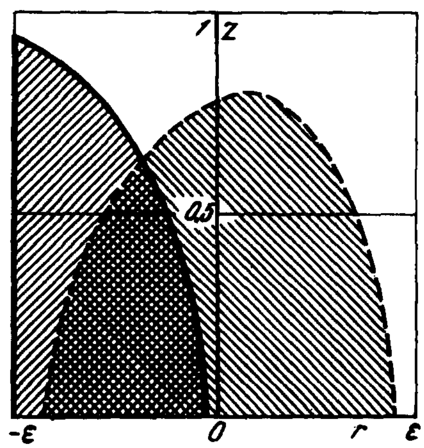

Fig. 2.

$$
\begin{aligned}
&\sigma_1=\frac{\nu}{1-\nu}\left(\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}\right)+\frac{\partial u_r}{\partial r}-\frac{\nu}{1-\nu}q_0,\quad \sigma_2=-q_0\\
&\sigma_3=\frac{\nu}{1-\nu}\left(\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z}\right)+\frac{\partial u_z}{\partial z}-\frac{\nu}{1-\nu}q_0,\quad q_0=q(a_0,0)\\
&\frac{\partial u_r}{\partial r}=\frac{\partial^2\Phi_0}{\partial r^2}+\frac{\lambda}{4(1-\nu)}\chi_1-\frac{1-2\nu}{2(1-\nu)}\chi_2\\
&\frac{\partial u_z}{\partial z}=\frac{\partial^2\Phi_0}{\partial z^2}+\frac{\lambda}{4(1-\nu)}\chi_3\\
&\frac{\partial^2\Phi_0}{\partial r^2}=-\frac{2(1-2\nu)}{\pi^2}\int_0^\infty\int_0^\infty\int_0^\infty E_1(p,t,\tau)K_*(p,t)dpdtd\tau\\
&\frac{\partial^2\Phi_0}{\partial z^2}=-\frac{2(1-2\nu)}{\pi^2}\int_0^\infty\int_0^\infty\int_0^\infty E_1(p,t,\tau)pK_{it}(\lambda t)dpdtd\tau \qquad (3.2)\\
&\chi_1=\sin\frac{\alpha}{2}\frac{\partial^2\Phi_1}{\partial r^2}-\cos\frac{\alpha}{2}\frac{\partial^2\Phi_2}{\partial r^2}=-\frac{4(1-\nu)}{\pi^2}\int_0^\infty\int_0^\infty E_2(p,t)K_*(p,t)dpdt
\end{aligned}
$$

$$\chi_2 = \sin\frac{\alpha}{2}\frac{\partial\Phi_1}{\partial r} - \cos\frac{\alpha}{2}\frac{\partial\Phi_2}{\partial r} = -\frac{4(1-\nu)}{\pi^2}\int_0^\infty\int_0^\infty E_2(p,t)p\,\mathrm{Re}\,K_{1+it}(\lambda p)dpdt$$

$$\chi_3 = \sin\frac{\alpha}{2}\frac{\partial^2\Phi_1}{\partial z^2} - \cos\frac{\alpha}{2}\frac{\partial^2\Phi_2}{\partial z^2} = -\frac{4(1-\nu)}{\pi^2}\int_0^\infty\int_0^\infty E_2(p,t)pK_{it}(\lambda p)dpdt$$

$$K_*(p,t) = pK_{it}(\lambda p) - (t\,\mathrm{Im}\,K_{1+it}(\lambda p) - \mathrm{Re}\,K_{1+it}(\lambda p))/\lambda$$

$$E_1(p,t,\tau) = \frac{\mathrm{sh}\,\pi t\,\mathrm{sh}\,\pi\tau}{\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi\tau}\left[W_+(\tau)\mathrm{cth}\frac{\alpha t}{2}E_+(\tau,p) - W_-(\tau)\mathrm{th}\frac{\alpha t}{2}E_-(\tau,p)\right]$$

$$E_2(p,t) = -2p\sin\alpha\,\mathrm{sh}\,\pi t\left[\frac{E_+(t,p)}{\mathrm{sh}\,\alpha t + t\sin\alpha} - \frac{E_-(t,p)}{\mathrm{sh}\,\alpha t - t\sin\alpha}\right]$$

$$E_\pm(t,p) = \frac{\Phi_\pm(t,p)}{\mathrm{ch}\,\pi t/2} + S(t,p),\quad S(t,p) = -\frac{1}{2\pi}\iint_\Omega q(r,z)K_{it}(p(r+b_0))\cos pz\,drdz$$

$$\Phi_\pm(t,p) = (1-2\nu)\int_0^\infty L_\pm(t,y)\left[\Phi_\pm(y,p) + \mathrm{ch}\frac{\pi y}{2}S(y,p)\right]dy,\quad 0 \leqslant t < \infty \tag{3.3}$$

Here σ_n ($n = 1, 2, 3$) are the principal stresses, u_r and u_z are the components of the displacement vector, $\Phi_n = \Phi_n(r, \varphi, z)$ ($n = 0, 1, 2$) are the functions which occur in the Papkovich–Neuber representation [3], and $b_0 = \lambda - a_0$.

To solve the Fredholm integral equations of the second kind (3.3), and also (1.3) and (1.12) the method of mechanical quadratures is employed using Gauss' quadrature formula.

If we put $\alpha = \pi$ in (3.1)–(3.3) and assume that the function $q(r, z)$ is defined in the elliptic region Ω by the relation

$$q(r,z) = q_0\sqrt{1-(r-a_0)^2/a_*^2 - z^2/b_*^2} \tag{3.4}$$

we obtain the following formula [9]

$$\sigma_e = (1-2\nu)q_0\sqrt{1-\beta+\beta^2}/(1+\beta) \tag{3.5}$$

where $\beta = b_*/a_*$ if $a_* \geqslant b_*$ and $\beta = a_*/b_*$ if $a_* \leqslant b_*$.

When deriving (3.5) we took into account the values of the integrals [10, No. 8.432.4 and No. 3.984.4]

$$\frac{2}{\pi^2}\int_0^\infty\int_0^\infty\int_0^\infty \frac{\mathrm{sh}\,\pi t\,\mathrm{sh}\,\pi\tau}{\mathrm{ch}\,\pi t + \mathrm{ch}\,\pi\tau}\left[\mathrm{cth}\frac{\pi\tau}{2}\mathrm{cth}\frac{\pi t}{2} + \mathrm{th}\frac{\pi\tau}{2}\mathrm{th}\frac{\pi t}{2}\right]K_*(p,t)\times$$

$$\times K_{i\tau}(p(r+b_0))\cos pz\,dpdtd\tau = \frac{d}{dz}\left[\frac{z}{(r-a_0)^2+z^2}\right] - 2\pi\delta(r-a_0)\delta(z) \tag{3.6}$$

$$\frac{1}{2\pi}\iint_\Omega q(r,z)\frac{d}{dz}\left[\frac{z}{(r-a_0)^2+z^2}\right]drdz = \frac{a_*q_0}{a_*+b_*}$$

In the second integral (3.6) the function $q(r, z)$ is defined by (3.4) and is the corresponding ellipse with centre at the point $r = a_0$, $z = 0$.

Table 2 shows values of the effective stresses $10^3\sigma_r$ at the point $r = a_0$, $\varphi = \alpha$, $z = 0$ as a function of $10^3\delta$ for different α and two orientations of the punch. Here $\nu = 0.3$, $\varepsilon = 0.15$, $\gamma = 0$, $\lambda = 0.35$, $A = 0.005$ and $B = 0.1$ up to values of $\alpha = 180°$ inclusive, or $\lambda = 0.15$, $A = 0.1$ and $B = 0.005$ below the row corresponding to $\alpha = 180°$. When $\alpha = 180°$, when the orientation of the punch and the value of λ play no role, the calculations were carried out using (3.5) in accordance with the exact solution of the problem [4]. The calculations show that, in the region of the edge, the dependence of σ_e on δ and also on λ may be non-monotonic. When the elliptic paraboloid approaches the edge along its semi-major axis, the values of σ_e are usually larger than when approaching along the semi-minor axis. It can be seen by comparing the first and third rows of Table 2 that when $\alpha = 110°$ more dangerous effective

Table 2

α, deg	$10^3\delta = 4$	4.5	5	5.5	6	6.5
65	1.08	1.35	1.33	1.36	1.33	1.61
90	1.48	1.45	1.70	1.74	1.71	2.06
110	1.48	1.70	1.74	1.75	1.96	1.69
135	1.21	1.44	1.44	1.74	1.31	0.924
180	1.17	1.24	1.31	1.37	1.43	1.49
135	1.03	1.08	1.15	1.21	1.26	1.33
110	0.952	1.02	1.08	1.15	1.21	1.28
90	0.832	0.894	0.943	0.996	1.05	1.10
65	0.501	0.525	0.549	0.573	0.595	0.610

stresses occur than when $\alpha = 65°$. As follows from (3.1)–(3.3), $\sigma_e \to \infty$ as $\lambda \to 0$, provided $q_0 \neq 0$ when $\lambda = 0$, i.e. contact is not broken off. If as $\lambda \to 0$ and for fairly acute angles α contact is broken off in the neighbourhood of the point where the punch and the wedge initially touch, we will have $\sigma_e = 0$ at this point [11].

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